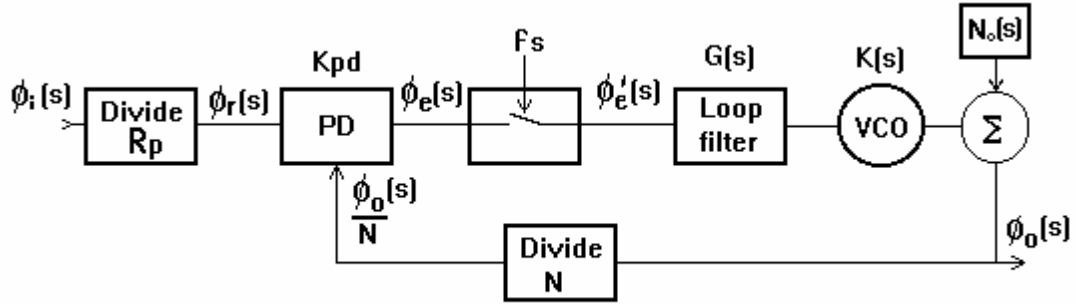


Analysis of Phase Noise in a Sampled PLL

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Block diagram of typical PLL



$$\phi_r(s) = \frac{\phi_i(s)}{R_p}$$

There is an argument that $\phi_o(s)/N$ and $\phi_r(s)$ should already be sampled by the action of the dividers, However this makes little difference as it is synchronously re-sampled by the sampler which leaves the result unchanged.

$$\phi_e(s) = \phi_r(s) - \frac{\phi_o(s)}{N}$$

Sampled version of ϕ_e is;

$$\phi_e'(s) = \left(\phi_r(s) - \frac{\phi_o(s)}{N} \right)'$$

$$\phi_e'(s) = \phi_r'(s) - \frac{\phi_o'(s)}{N}$$

$$\phi_o(s) = \phi_e'(s) \cdot GK(s) + N_o(s)$$

where

$$GK(s) = G(s) \cdot K(s)$$

$$\phi_e'(s) = \left(\phi_r'(s) - \frac{\phi_e'(s) \cdot GK(s) + N_o(s)}{N} \right)'$$

$$\phi_e'(s) = \phi_r'(s) - \phi_e'(s) \cdot \frac{GK(s)}{N}$$

Then with a little rearrangement we get the output phase as a function of the input noise

$$\phi_o(s) = \left(\phi_r'(s) - \frac{\phi_o'(s)}{N} \right) \cdot GK(s) + N_o(s)$$

$$\phi_o(s) = \phi_r'(s) \cdot GK(s) - \frac{\phi_o'(s)}{N} \cdot GK(s) + N_o(s)$$

$$\phi_o'(s) = \phi_r'(s) \cdot GK(s) - \frac{\phi_o'(s)}{N} \cdot GK(s) + N_o'(s)$$

$$\phi_o'(s) \cdot \left(1 + \frac{GK(s)}{N} \right) = \phi_r'(s) \cdot GK(s) + N_o'(s)$$

$$\phi_o'(s) = \frac{\phi_r'(s) \cdot GK(s)}{\left(1 + \frac{GK(s)}{N} \right)} + \frac{N_o'(s)}{\left(1 + \frac{GK(s)}{N} \right)}$$

Putting $\phi_o'(s)$ into the expression for $\phi_o(s)$ we get:

$$\phi_o(s) = \phi_r'(s) \cdot GK(s) - \frac{\frac{\phi_r'(s) \cdot GK(s)}{\left(1 + \frac{GK(s)}{N} \right)} + \frac{N_o'(s)}{\left(1 + \frac{GK(s)}{N} \right)}}{N} \cdot GK(s) + N_o(s)$$

Let us separate out the noise at the output due to reference noise and VCO noise.

First we take the noise due to the reference:

$$\phi_{or}(s) = \phi_r'(s) \cdot GK(s) - \frac{\phi_r'(s) \cdot GK(s)}{\left(1 + \frac{GK(s)}{N} \right)} \cdot \frac{GK(s)}{N}$$

$\phi_{or}(s)$ is the output phase due to the reference

$$\begin{aligned} &= \phi_r'(s) \cdot GK(s) \cdot \left[1 - \frac{GK(s)}{N \cdot \left(1 + \frac{GK(s)}{N} \right)} \right] \\ &= \frac{\phi_r'(s) \cdot GK(s)}{N \cdot \left(1 + \frac{GK(s)}{N} \right)} \cdot \left[N \cdot \left(1 + \frac{GK(s)}{N} \right) - GK(s) \right] \\ &= \frac{\phi_r'(s) \cdot GK(s)}{N \cdot \left(1 + \frac{GK(s)}{N} \right)} \cdot (N + GK(s) - GK(s)) \end{aligned}$$

$$\phi_{Or}(s) = \frac{\phi_r(s) \cdot GK(s)}{\left(1 + \frac{GK(s)}{N}\right)}$$

This is effectively the sampled reference noise modified by the transfer response of the loop.
Now let us look at the contribution to ϕ_o due to VCO noise $N_o(s)$

$$\begin{aligned}\phi_{oN_o}(s) &= \frac{-\phi_o(s)}{N} \cdot GK(s) + N_o(s) \\ &= \frac{-N'_o(s)}{\left(1 + \frac{GK(s)}{N}\right)} \cdot \frac{GK(s)}{N} + N_o(s) \\ &= N_o(s) - N'_o(s) \cdot \frac{Tr'(s)}{N}\end{aligned}$$

where

$$Tr'(s) = \frac{GK(s)}{\left(1 + \frac{GK(s)}{N}\right)}$$

Now we need to develop the maths.
Call the impulse sampled $f(t)$, $f'(t)$

$$f'(t) = f(t) \cdot \delta(t)$$

$$f(t) \cdot \delta(t) = \sum_{n=-\infty}^{\infty} f(t) \cdot \delta(t - n \cdot T)$$

where

$$T = \frac{1}{F_s}$$

this may be represented by the fourier series

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{j \cdot \omega_s \cdot n \cdot t}$$

where

$$\omega_s = 2 \cdot \frac{\pi}{T}$$

$$C_n = \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t) \cdot e^{-j \cdot \omega \cdot t} dt$$

$$= \frac{1}{T}$$

So let us now call $F'(s)$ the laplace transform of the sampled time function $f'(t)$

$$\begin{aligned} F'(s) &= L(f'(t)) \\ &= \frac{1}{T} \cdot \sum_{n=-\infty}^{\infty} \int_0^{\infty} f(t) \cdot e^{-(s-j\omega_s \cdot n) \cdot t} dt \\ &= \frac{1}{T} \cdot \sum_{n=-\infty}^{\infty} F(s - j\omega_s \cdot n) \end{aligned}$$

Now let us apply this to the reference noise:

$$\begin{aligned} \phi_{Or}(s) &= \frac{\phi_r(s) \cdot GK(s)}{\left(1 + \frac{GK(s)}{N}\right)} \\ &= \phi_r(s) \cdot \frac{GK(s)}{1 + \frac{1}{N} \cdot \sum_{n=-\infty}^{\infty} GK(s - j\omega_s \cdot n)} \end{aligned}$$

Where

$$\phi_r(s) = \frac{\phi_i(s)}{Rp}$$

Note the sampled function has been multiplied by a factor of T. This is because in the design this is taken into account by the phase detector.

And for the noise due to the VCO

$$\phi_{oN_o}(s) = N_o(s) - N'_o(s) \cdot \frac{Tr'(s)}{N}$$

where

$$\begin{aligned} Tr'(s) &= \frac{GK(s)}{1 + \frac{GK(s)}{N}} \\ &= N_o(s) - \sum_{n=-\infty}^{\infty} N_o(s - j\omega_s \cdot n) \cdot \frac{1}{N} \cdot \frac{GK(s)}{\left(1 + \frac{1}{N} \cdot \sum_{n=-\infty}^{\infty} GK(s - j\omega_s \cdot n)\right)} \end{aligned}$$

Now take out the $n = 0$ term and add this to the sum of the rest.

For $n = 0$

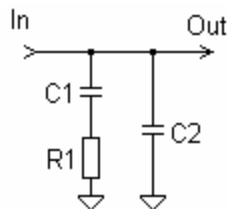
$$\begin{aligned}
\phi_{oN_o}(s) &= N_o(s) - N_o(s) \cdot \frac{1}{N} \cdot \frac{GK(s)}{1 + \frac{GK(s)}{N}} \\
&= N_o(s) \cdot \left(1 - \frac{1}{N} \cdot \frac{GK(s)}{1 + \frac{GK(s)}{N}} \right) \\
&= \frac{N_o(s)}{\left(1 + \frac{GK(s)}{N} \right)} \cdot \left(1 + \frac{GK(s)}{N} - \frac{1}{N} \cdot GK(s) \right) \\
&= \frac{N_o(s)}{\left(1 + \frac{GK(s)}{N} \right)}
\end{aligned}$$

So combining the two we get:

$$\begin{aligned}
\phi_{oN_o}(s) &= \frac{N_o(s)}{\left(1 + \frac{GK(s)}{N} \right)} + \sum_{n=-\infty}^{-1} N_o(s - j \cdot \omega_s \cdot n) \cdot \frac{1}{N} \cdot \frac{GK(s)}{\left(1 + \frac{1}{N} \cdot \sum_{n=-\infty}^{-1} GK(s - j \cdot \omega_s \cdot n) \right)} \dots \\
&\quad + \sum_{n=1}^{\infty} N_o(s - j \cdot \omega_s \cdot n) \cdot \frac{1}{N} \cdot \frac{GK(s)}{\left(1 + \frac{1}{N} \cdot \sum_{n=1}^{\infty} GK(s - j \cdot \omega_s \cdot n) \right)}
\end{aligned}$$

Now we can put in some component values that correspond to those used in the test PLL and observe the results.

Loop filter values for a standard charge pump type phase detector



$$R1 := 10 \cdot 10^3$$

$$C1 := 18 \cdot 10^{-9}$$

$$C2 := 2.7 \cdot 10^{-9}$$

Some more values for loop components and sampling frequency

Phase detector gain for charge pump using 4mA, K_{pd} :

$$K_{pd} := \frac{4 \cdot 10^{-3}}{2 \cdot \pi}$$

Phase detector comparison frequency of 25kHz (sampling frequency f_s)

$$f_s := 25 \cdot 10^3$$

Reference divider ratio, R_p :

$$R_p := 672$$

Sampling frequency in radians ω_s

$$\omega_s := 2 \cdot \pi \cdot f_s$$

VCO gain in radians/second/volt (for a VCO with a tuning sensitivity of 20MHz/Volt), $K_{\omega vco}$:

$$K_{\omega vco} := 20 \cdot 10^6 \cdot 2 \cdot \pi$$

Loop division ratio, N :

$$N := 35200$$

$G(s)$, which includes phase detector gain and loop filter response, becomes.

$$G(s) := \frac{\left(R1 + \frac{1}{s \cdot C1} \right) \cdot \frac{1}{s \cdot C2}}{R1 + \frac{1}{s \cdot C1} + \frac{1}{s \cdot C2}} \cdot K_{pd}$$

Which simplifies to

$$G(s) := \frac{(R1 \cdot s \cdot C1 + 1) \cdot K_{pd}}{[s \cdot (R1 \cdot s \cdot C1 \cdot C2 + C2 + C1)]}$$

And $K(s)$ is given by:

$$K(s) := \frac{K_{\omega vco}}{s}$$

So the overall forward gain, $GK(s)$, the combination of oscillator gain and frequency response, the phase detector gain and loop filter frequency response is.

$$GK(s) := G(s) \cdot K(s)$$

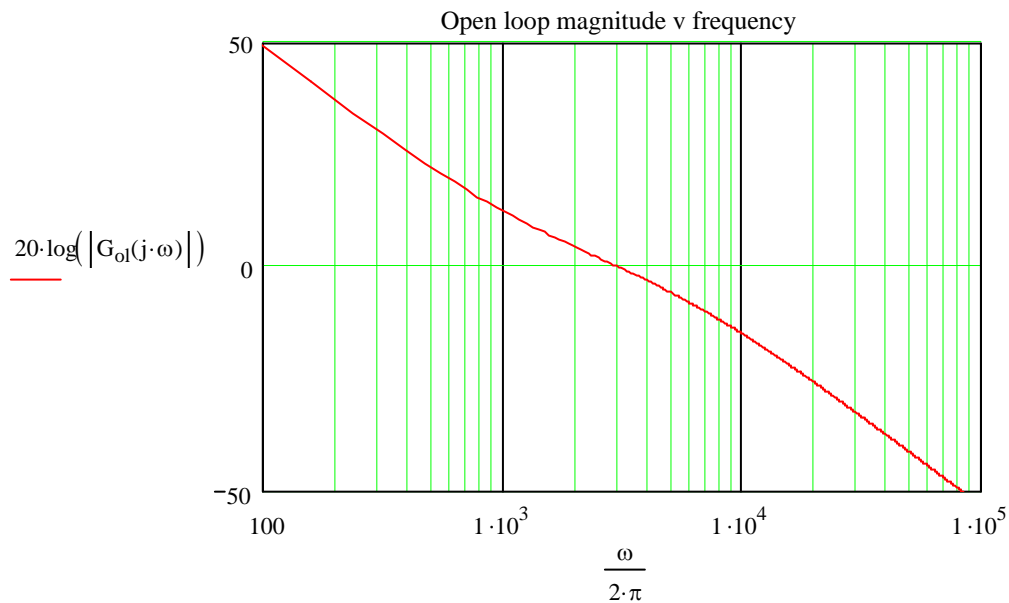
Continuous system

Now let us look at the results for a continuous system so that we can compare them later with the results of the sampled system.

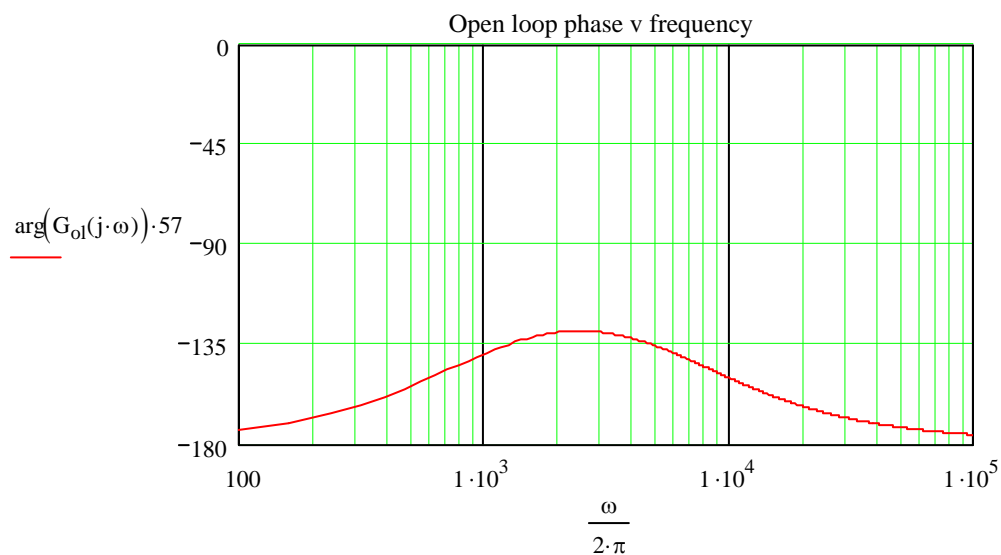
Open loop gain for a continuous system is:

$$G_{ol}(s) := \frac{GK(s)}{N}$$

Plot of open loop gain for continuous system with open loop bandwidth set to ~3kHz



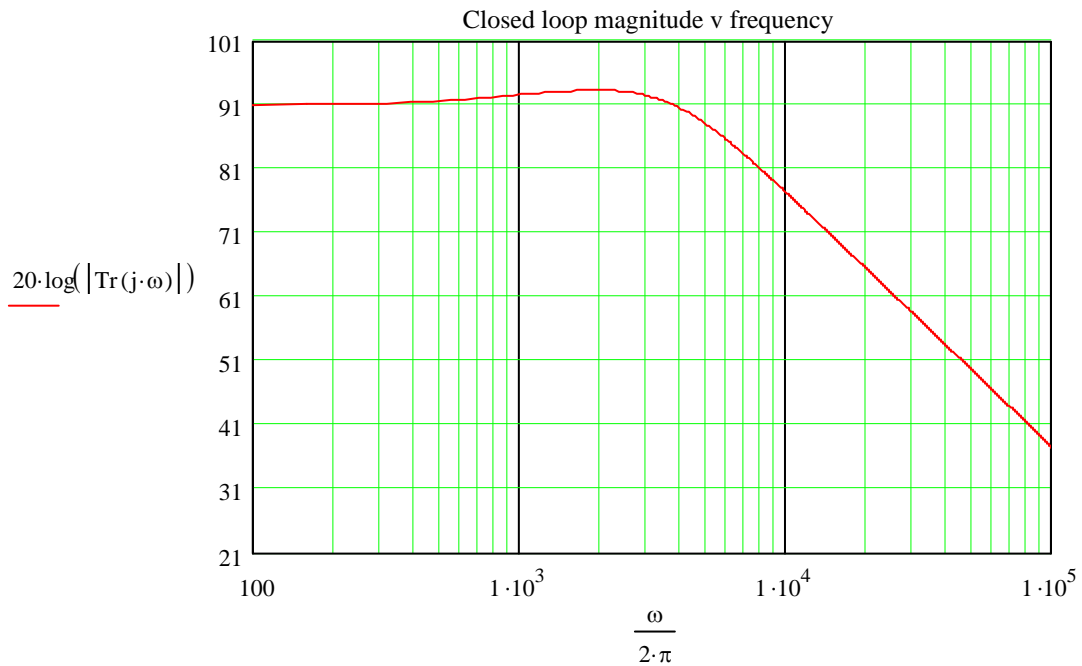
Plot of open loop phase for continuous system



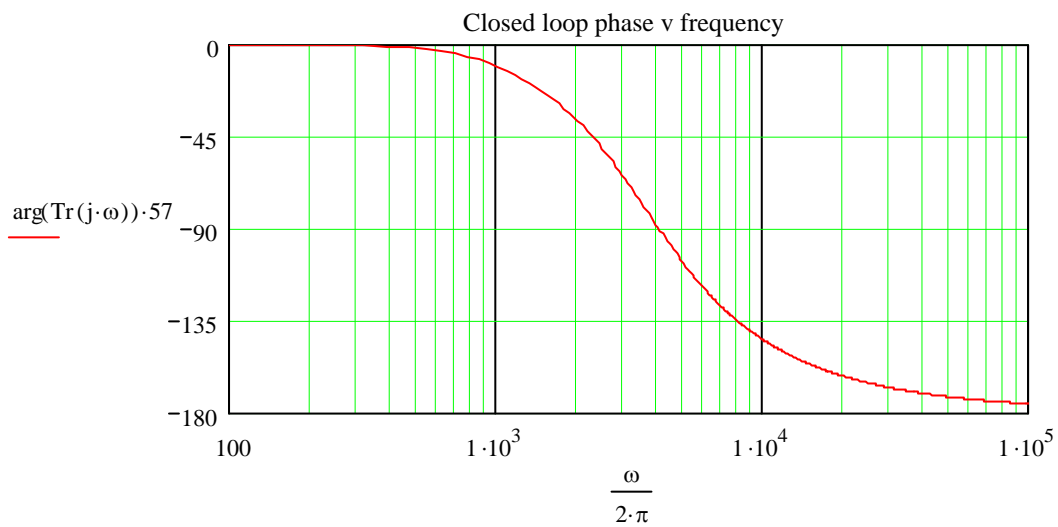
The closed loop transfer response is given by:

$$Tr(s) := \frac{GK(s)}{1 + \frac{GK(s)}{N}}$$

Which when plotted gives the familiar response shown below



And the familiar phase response



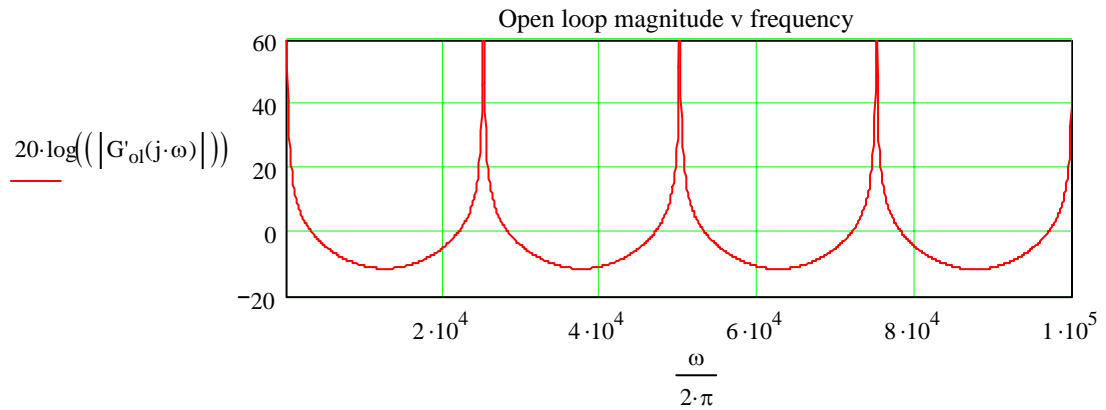
Sampled system

Open loop gain for a sampled system is given by:

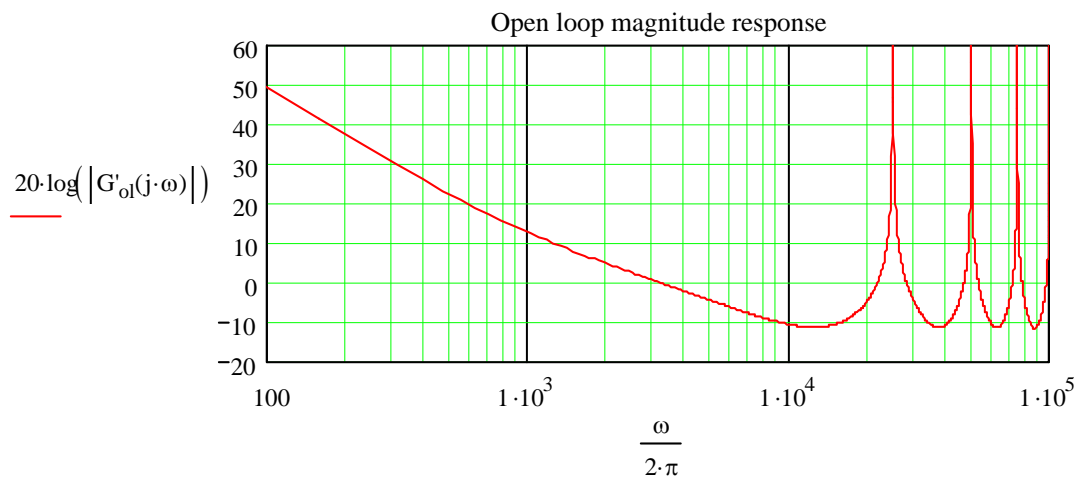
$$G'_{ol}(s) := \frac{1}{N} \cdot \sum_{n=-5}^5 \text{GK}(s - j \cdot n \cdot \omega_s)$$

Note the gain has been scaled by a factor of T to keep the value correct. Also n has been reduced from infinity to 5 to speed the calculations

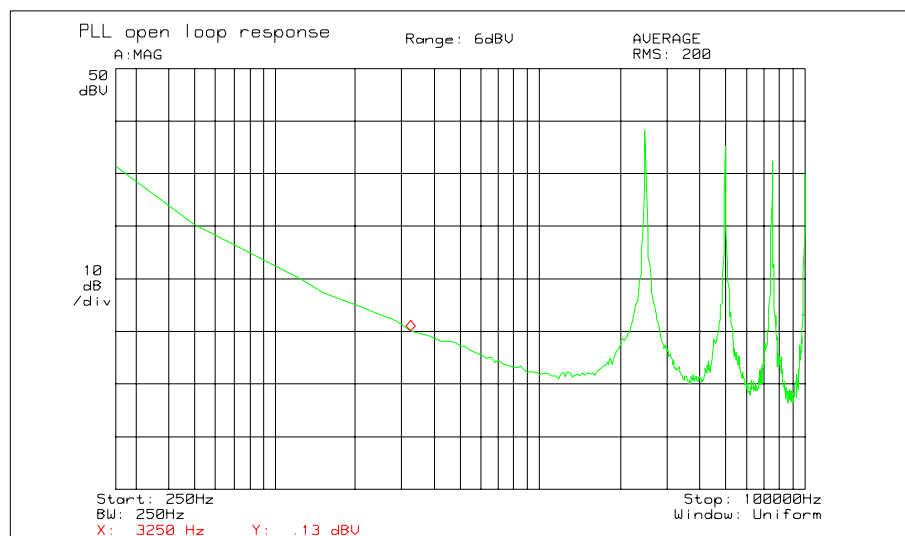
This is plotted below on a linear scale. Notice how the open loop gain is aliased about each harmonic of the sampling frequency



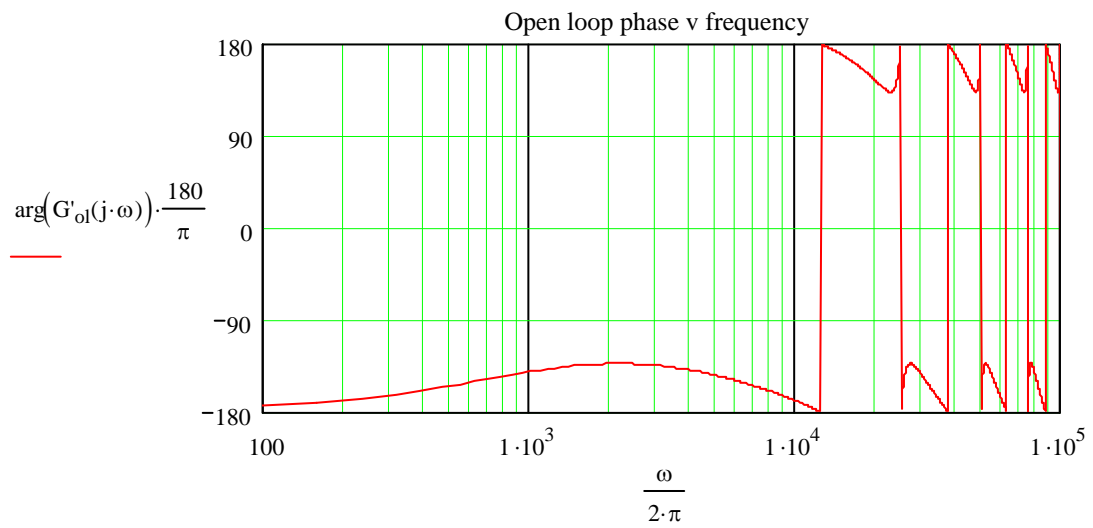
The same information is plotted on the more familiar log scale so that the gain and phase margin can be more easily seen.



Plot shows the measured open loop gain of the test PLL



Open loop phase with sampling

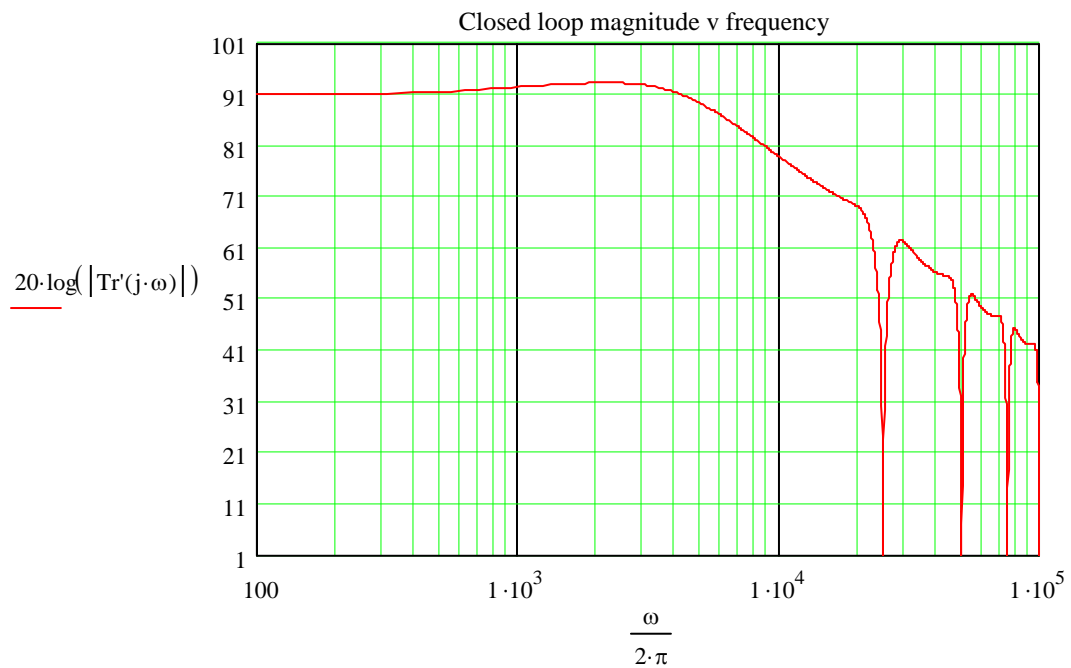


Notice, that compared to the continuous time system, the open loop gain passes through unity gain at a higher frequency and that the phase margin is reduced at this frequency.

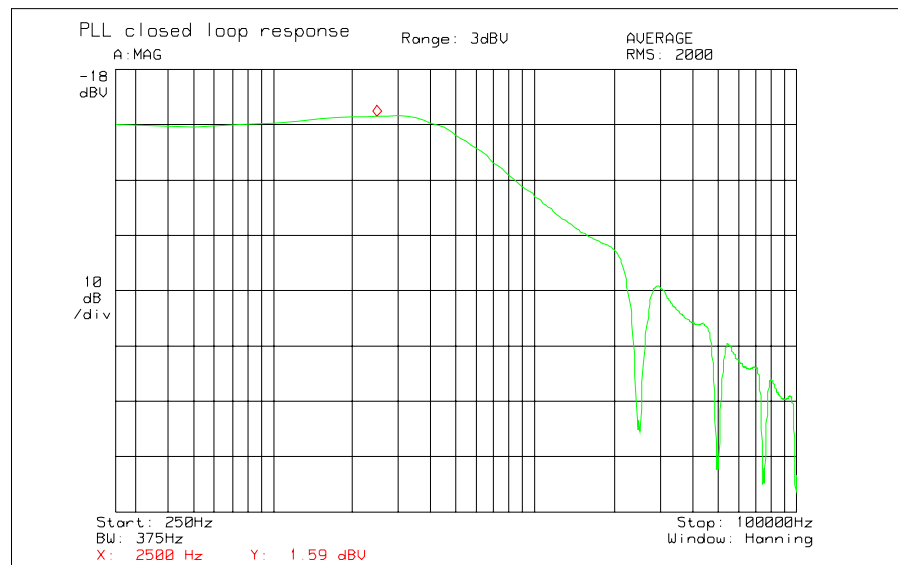
The corresponding closed loop transfer response with sampling, $Tr'(s)$, is given by
 Note this is not a true transfer response as it is the output phase divided by the *sampled* input phase

$$Tr'(s) := \frac{GK(s)}{1 + \frac{1}{N} \cdot \sum_{n=-5}^5 GK(s - j \cdot n \cdot \omega_s)}$$

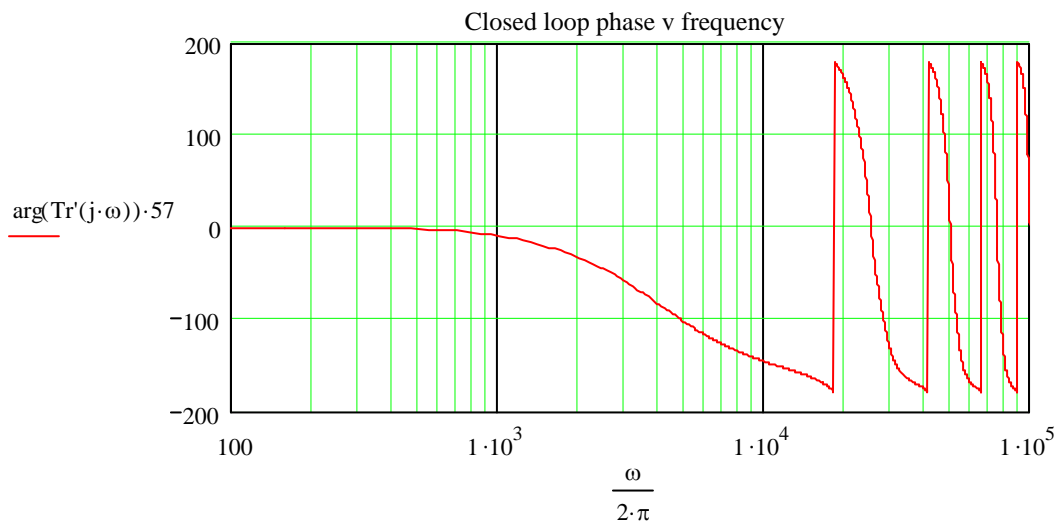
Plot of the closed loop transfer response with sampling



Plot below shows the closed loop response of the test PLL



Plot of closed loop phase with sampling



At this point the functions are redefined as a function of sampling frequency so that the effect of different sampling frequencies may be observed later.

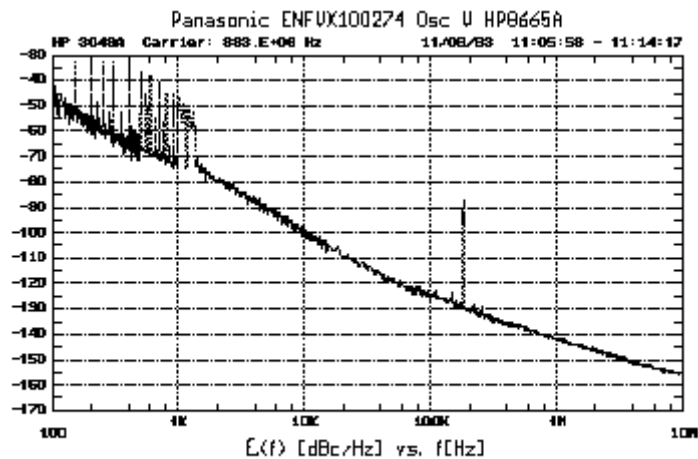
$$G_{ol}(s, f_s) := \frac{1}{N} \cdot \sum_{n=-10}^{10} GK(s - j \cdot n \cdot 2 \cdot \pi \cdot f_s)$$

$$Tr(s, f_s) := \frac{GK(s)}{1 + \frac{1}{N} \cdot \sum_{n=-10}^{10} GK(s - j \cdot n \cdot 2 \cdot \pi \cdot f_s)}$$

Phase Noise in a sampled loop

Now let us look at the effect of sampling on the phase noise of the synthesiser. The main sources of noise are, Free running VCO noise, "Phase detector noise", thermal noise in the loop filter components and reference oscillator noise. (Divider noise is lumped in with phase detector noise.)

VCO Phase noise:



Let us define the segments

Let us call the SSB phase noise in the far-off region, where it has an f^0 profile, L_0 and the offset f_0 , similarly in the region where the phase noise has an f^{-2} profile, L_2 and the offset f_2 , and finally the region where it has an f^{-3} profile L_3 and the offset f_3 .

Then for a typical oscillator such as the one measured above:

$$\begin{aligned} L_0 &:= -155 & f_0 &:= 3 \cdot 10^6 \\ L_2 &:= -126 & f_2 &:= 100 \cdot 10^3 \\ L_3 &:= -70 & f_3 &:= 1 \cdot 10^3 \end{aligned}$$

The f^{-3} section of the noise due to $1/f$ noise in the oscillator circuit is modelled by:

$$L_3(s) := 10^{\frac{L_3}{10}} \cdot f_3^3 \cdot \left(\frac{s}{2 \cdot \pi} \right)^{-3}$$

The f^{-2} section due to peaking of white noise by oscillator is modelled by

$$L_2(s) := 10^{\left(\frac{L_2}{10} \right)} \cdot f_2^2 \cdot \left(\frac{s}{2 \cdot \pi} \right)^{-2}$$

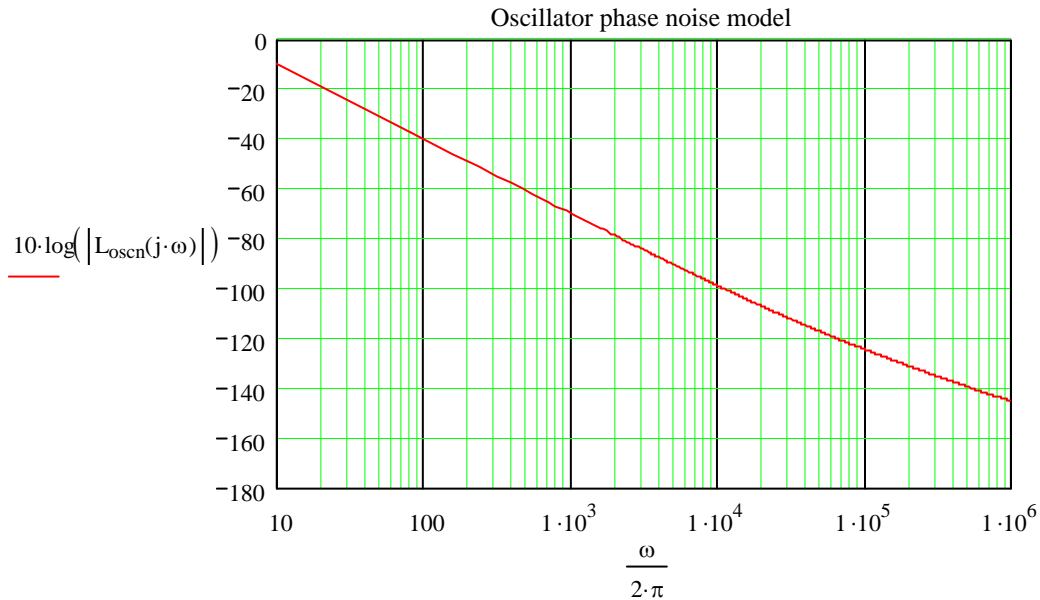
And the broad-band white noise due to thermal noise and NF of the active device is modelled by:

$$L_0(s) := 10^{\left(\frac{L_0}{10} \right)} \cdot f_0^0 \cdot \left(\frac{s}{2 \cdot \pi} \right)^0$$

Combining these contributions we get an overall noise model of the oscillator, $L_{\text{oscn}}(s)$

$$L_{\text{oscn}}(s) := |L_0(s)| + |L_2(s)| + |L_3(s)|$$

Plot of modelled noise of the free running VCO



The phase noise at the output of the loop is made up of the noise when $n = 0$ and the sum of the output noise, divided by N , which appears as an error signal at the phase detector and is acted on by the equivalent transfer function.

Phase noise due to the loops action on the $n = 0$ component of oscillator noise

$$L_{\text{osc1}}(s) := \frac{L_{\text{oscn}}(s)}{(1 + G_{\text{ol}}(s))^2}$$

Phase noise due to the loops action on the $n \neq 0$ component of oscillator noise

$$L_{\text{osc2}}(s, f_s) := \left(\frac{|Tr'(s, f_s)|}{N} \right)^2 \cdot \frac{L'_{\text{oscn1}}(s, f_s)}{(|1 + G'_{\text{ol}}(s, f_s)|)^2}$$

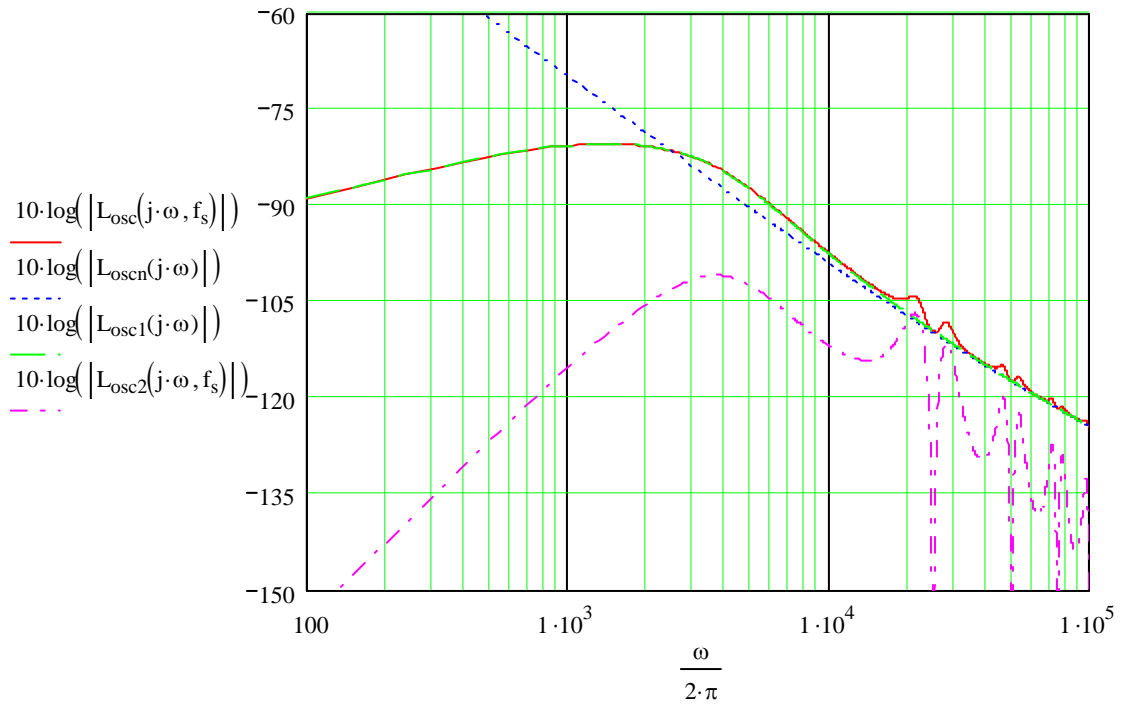
Where the noise of oscillator after sampling L'_{oscn1} is given by:

$$L'_{\text{oscn1}}(s, f_s) := \sum_{n=-5}^{-1} L_{\text{oscn}}(s - j \cdot \omega_s \cdot n) + \sum_{n=1}^5 L_{\text{oscn}}(s - j \cdot \omega_s \cdot n)$$

The overall noise due to the oscillator in the loop is the sum of the two components of oscillator noise

$$L_{\text{osc}}(s, f_s) := |L_{\text{osc2}}(s, f_s)| + |L_{\text{osc1}}(s)|$$

We can now plot these various components;



L_{osc} is the total contribution to the overall noise after the action of the loop

L_{oscn} is the noise of the free running oscillator.

L_{osc1} is the noise due to the $n = 0$ component

L_{osc2} is the noise due to $n \neq 0$ components

Thermal noise due to the real part of the loop filter impedance

Thermal noise, due to the real part of the impedance of the loop filter, modulates the VCO to give phase noise at the output of the VCO.

$$k := 1.38 \cdot 10^{-23} \quad \text{Boltzman's constant}$$

$$T := 290 \quad \text{Absolute temperature}$$

From FM theory we get the SSB noise due to modulation by thermal noise is given by:

$$L_{mod}(s) := \left(\frac{K_{\omega vco}}{2 \cdot \pi} \right)^2 \cdot 8 \cdot k \cdot T \cdot \frac{|\text{Re}(Z(s))|}{s \cdot s \cdot \left(2 \cdot \frac{1}{2 \cdot \pi} \right)^2}$$

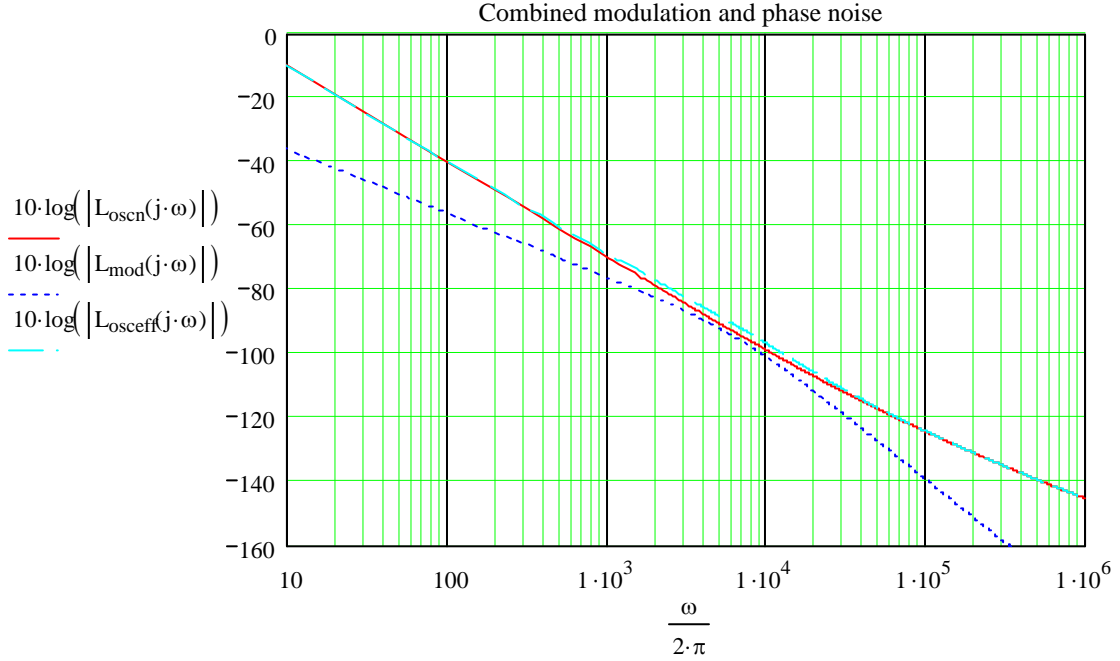
Where $Z(s)$ is the impedance of the loop filter

Which simplifies to:

$$L_{mod}(s) := \left(2 \cdot K_{\omega vco}^2 \cdot k \cdot T \right) \cdot \left| \frac{\text{Re}(Z(s))}{s \cdot s} \right|$$

This noise combines with the VCO phase noise before being modified by the loop

$$L_{osceff}(s) := |L_{mod}(s)| + |L_{oscn}(s)|$$



Sampled version of the combined VCO noise and modulation noise.

$$L'_{\text{osceff}}(s, f_s) := \sum_{n=-5}^5 L_{\text{osceff}}(s - j \cdot \omega_s \cdot n)$$

Contribution to noise due to the sampling process. ie. the total noise excluding the noise of the continuous system

Phase noise due to the loops action on the $n = 0$ component of oscillator noise

$$L_{\text{osceff1}}(s) := \frac{L_{\text{osceff}}(s)}{(|1 + G_{\text{ol}}(s)|)^2}$$

Phase noise due to the loops action on the $n \neq 0$ component of oscillator noise

$$L_{\text{osceff2}}(s, f_s) := \left(\frac{|\text{Tr}(s, f_s)|}{N} \right)^2 \cdot \frac{L'_{\text{osceff1}}(s, f_s)}{(|1 + G'_{\text{ol}}(s, f_s)|)^2}$$

Where $L'_{\text{osceff1}}(s, f_s)$ is given by:

$$L'_{\text{osceff1}}(s, f_s) := \sum_{n=-5}^{-1} L_{\text{osceff}}(s - j \cdot \omega_s \cdot n) + \sum_{n=1}^5 L_{\text{osceff}}(s - j \cdot \omega_s \cdot n)$$

The overall noise, $L_{\text{osctot}}(s, f_s)$, due to the oscillator and the noise due to modulation of the oscillator by thermal noise in the loop filter components is the sum of the two components of oscillator noise.

$$L_{\text{osctot}}(s, f_s) := |L_{\text{osceff2}}(s, f_s)| + |L_{\text{osceff1}}(s)|$$

Phase detector noise.

Here the term phase detector noise is used to cover noise due to the charge pumps, reference dividers and programmable dividers as it is not possible to isolate the cause in many of the single chip PLLs. For any given sampling frequency the effect they have on the loop is essentially the same irrespective of the cause.

The phase detector noise for any given chip at a particular comparison frequency is usually determined from measurement. Typically for the National Semiconductor LMX1511 chip used in the test PLL, the value is equivalent to $\sim -163\text{dBc/Hz}$ at a comparison frequency of 25kHz. Often the measure termed "Figure of Merit" is used by manufacturers where the noise is referenced back to 1Hz. From this the noise can be predicted for any sampling frequency. This relationship usually holds over a wide range of frequencies but may eventually break down at higher frequencies.

Some suggest that the noise gets worse with frequency because the charge pumps are on for longer compared to the sampling period and therefore allow more noise to be transferred to the loop filter. In fact the noise follows exactly what would be expected from FM theory and sampling theory. From FM theory we would expect the noise to increase by 6dB for every doubling of frequency ($20 \log f_s$), however from sampling theory we would expect the noise power per Hz to decrease by 3dB for every doubling of the sampling frequency ($-10 \log f_s$) as the noise power is now spread over twice the bandwidth. The net result is an increase in the phase noise by 3dB for every doubling of the sampling frequency ($10 \log f_s$).

So let us take the phase noise at the phase detector due to charge pumps or dividers etc., referenced back to 1Hz to be:

$$L_{\text{pd}_1\text{Hz}} := -207 \text{ dBc/Hz}$$

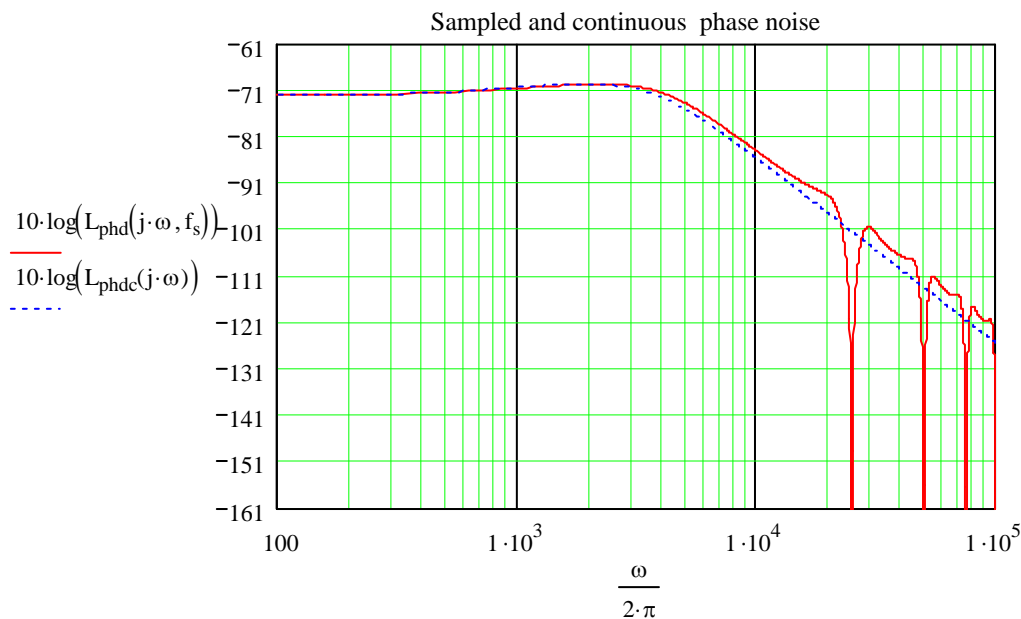
$$L_{\text{pd}} := 10 \cdot \log(f_s) + L_{\text{pd}_1\text{Hz}}$$

$$L_{\text{pd}} = -163.021$$

This is acted on by the loop to give a contribution to the overall noise of:

$$L_{\text{phd}}(s, f_s) := 10^{10} \cdot \left(\frac{L_{\text{pd}}}{|\text{Tr}(s, f_s)|} \right)^2$$

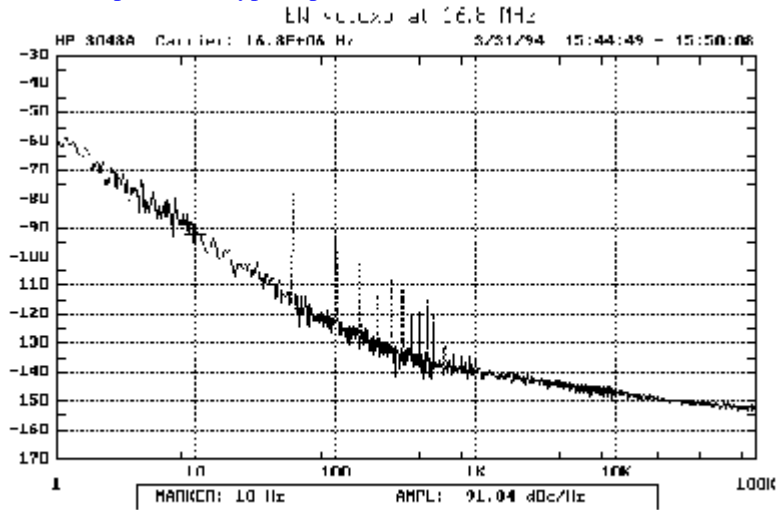
The plot below shows the difference between the output noise due to the phase detector for a continuous system and for a sampled system. Note that there is a significant difference between the two. The apparent bandwidth for the sampled system is higher, and the noise at higher frequency offsets is 3 to 4 dB higher except at multiples of the sampling frequency where the contribution falls to zero.



Reference oscillator noise

The reference oscillator may also contribute to the overall phase noise close to the carrier. The SSB phase noise of the reference oscillator appears at the output of the synthesiser multiplied by the overall division ratio between the reference and the output frequency.

Below is a plot of the typical phase noise of the 16.8MHz VCTCXO used in the test PLL.



Typically, for the type of VCTCXOs used, the phase noise is flat from large frequency offsets in to about 100kHz, then rises at ~ 10dB/decade to ~ 400Hz from the carrier, then it rises at ~30dB/decade. Reference oscillator phase noise may be modeled by defining the noise at three spot points and interpolating between. The points are 1MHz, 10kHz, and 10Hz

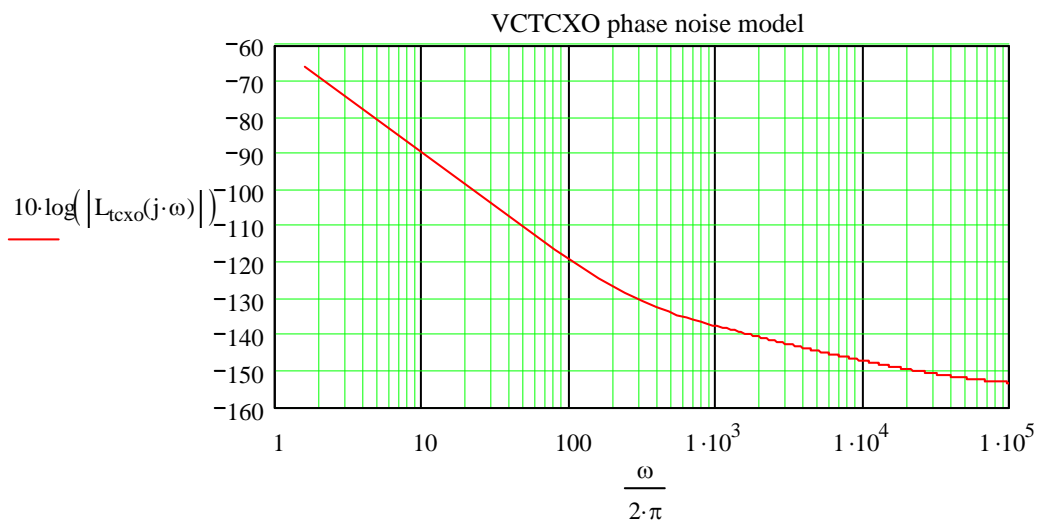
$$L_{x0} := -155 \quad f_{x0} := 1 \cdot 10^6$$

$$L_{x1} := -148 \quad f_{x1} := 10 \cdot 10^3$$

$$L_{x3} := -90 \quad f_{x3} := 10$$

Then reference oscillator phase noise is modeled by:

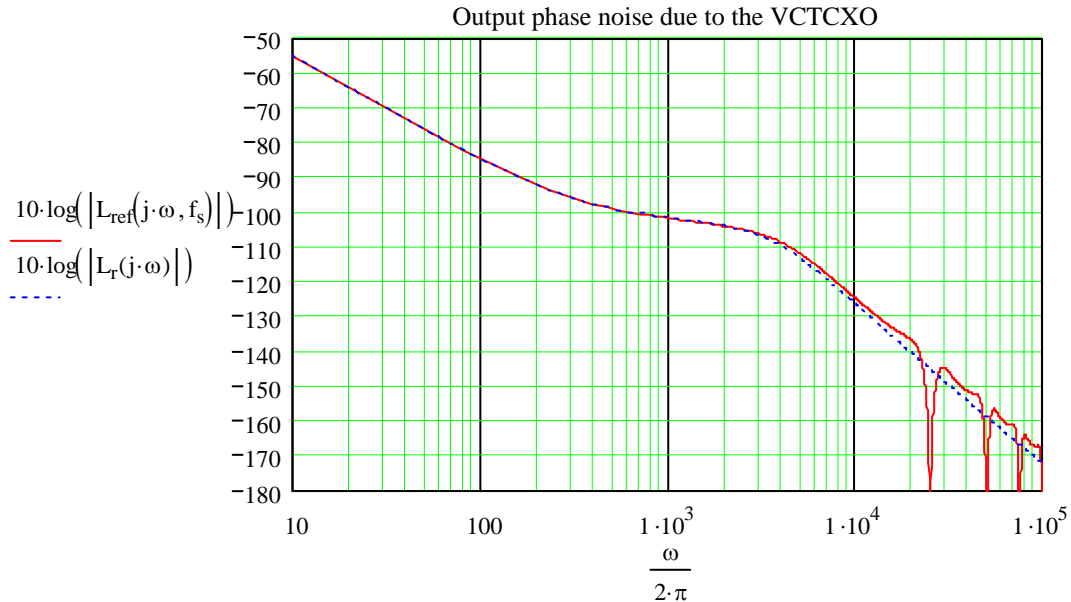
$$L_{\text{tcxo}}(s) := \left| 10 \frac{L_{x1}}{10} \cdot f_{x1} \cdot \left(\frac{s}{2 \cdot \pi} \right)^{-1} \right| + \left| 10 \frac{L_{x3}}{10} \cdot f_{x3}^3 \cdot \left(\frac{s}{2 \cdot \pi} \right)^{-3} \right| + \left| \frac{L_{x0}}{10} \right|$$



Now we can plot the effect the loop has on the phase noise of the reference oscillator. Usually the reference noise is well below the contribution due to the phase detector or dividers except at low offset frequencies. (Typically below a few hundred Hz)

$$L_T(s) := L_{\text{tcxo}}(s) \cdot \left(\frac{1}{R_p}\right)^2 \cdot (|\text{Tr}(s)|)^2$$

$$L_{\text{ref}}(s, f_s) := L_{\text{tcxo}}(s) \cdot \left(\frac{1}{R_p}\right)^2 \cdot (|\text{Tr}(s, f_s)|)^2$$



Combining all the noise sources

Each of these sources is modified by the PLL according to the equations below
Noise generated by modulation of the VCO by thermal noise in the loop filter.

$$L_{\text{mod}}(s) := \left(\frac{K_{\omega\text{vco}}}{2 \cdot \pi}\right)^2 \cdot 8 \cdot k \cdot T \cdot \frac{\text{Re}(Z(s))}{s \cdot s \cdot \left(2 \cdot \frac{1}{2 \cdot \pi}\right)^2}$$

Noise due to the phase detector modified by the loop response.

$$L_{\text{phd}}(s, f_s) := 10^{10} \cdot \frac{L_{\text{pd}}}{(|\text{Tr}(s, f_s)|)^2}$$

Free running VCO noise modified by the loop

$$L_{\text{osctot}}(s, f_s) := |L_{\text{osceff2}}(s, f_s)| + |L_{\text{osceff1}}(s)|$$

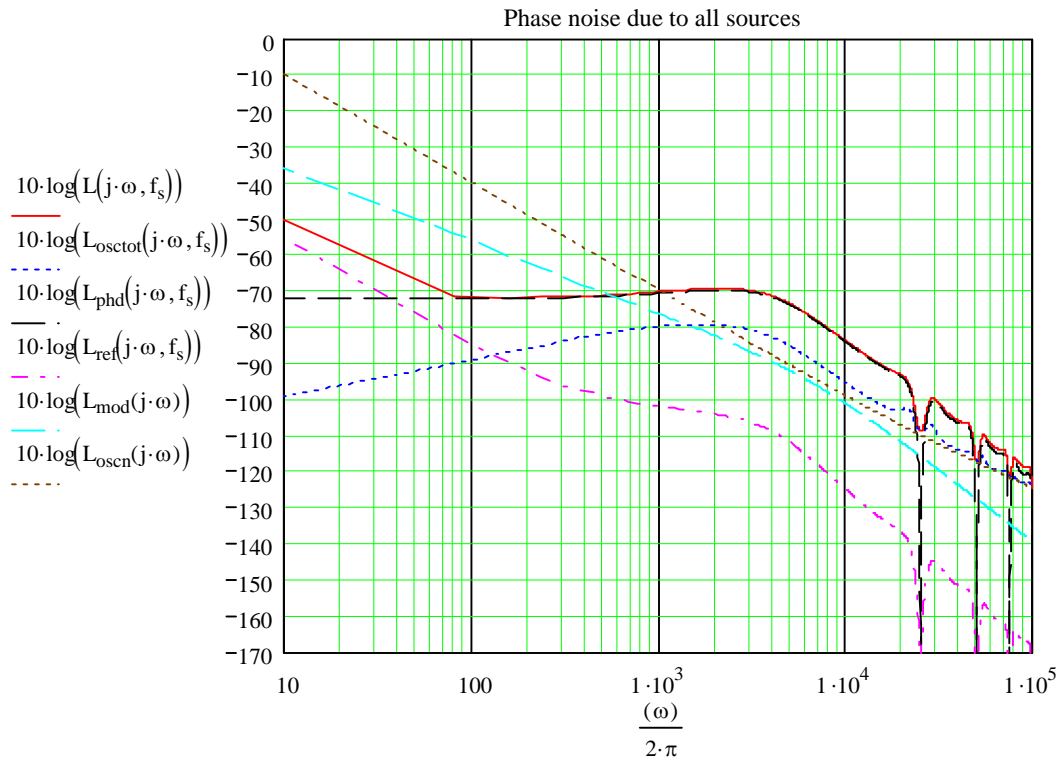
Reference oscillator noise modified by the loop

$$L_{\text{ref}}(s, f_s) := L_{\text{tcxo}}(s) \cdot \left(\frac{1}{R_p}\right)^2 \cdot (|\text{Tr}(s, f_s)|)^2$$

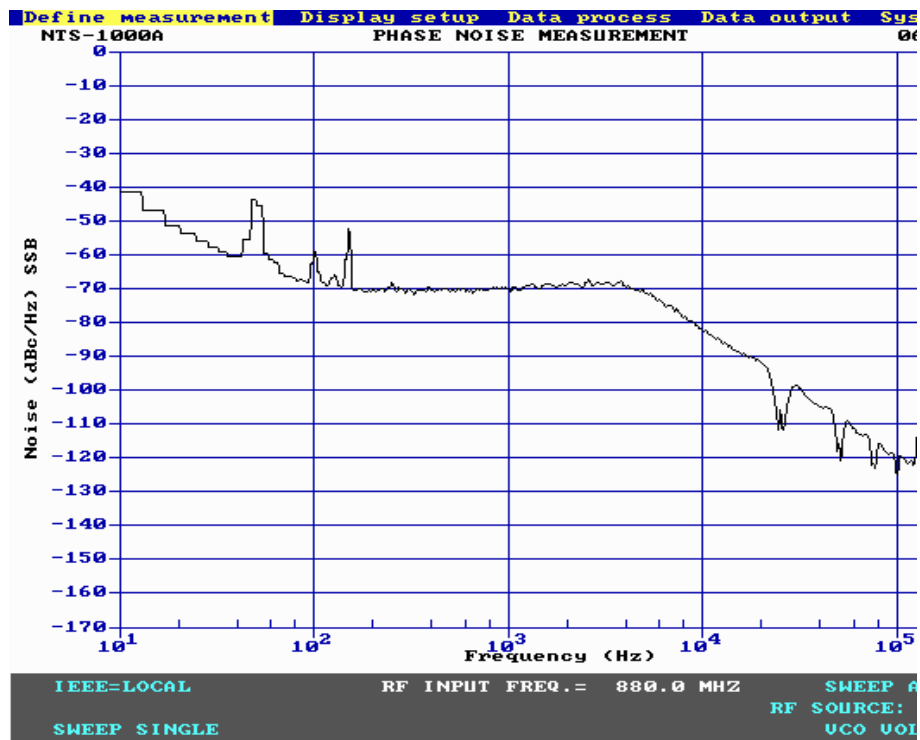
Combined output phase noise due to all the sources above

$$L(s, f_s) := L_{\text{phd}}(s, f_s) + L_{\text{osctot}}(s, f_s) + L_{\text{ref}}(s, f_s)$$

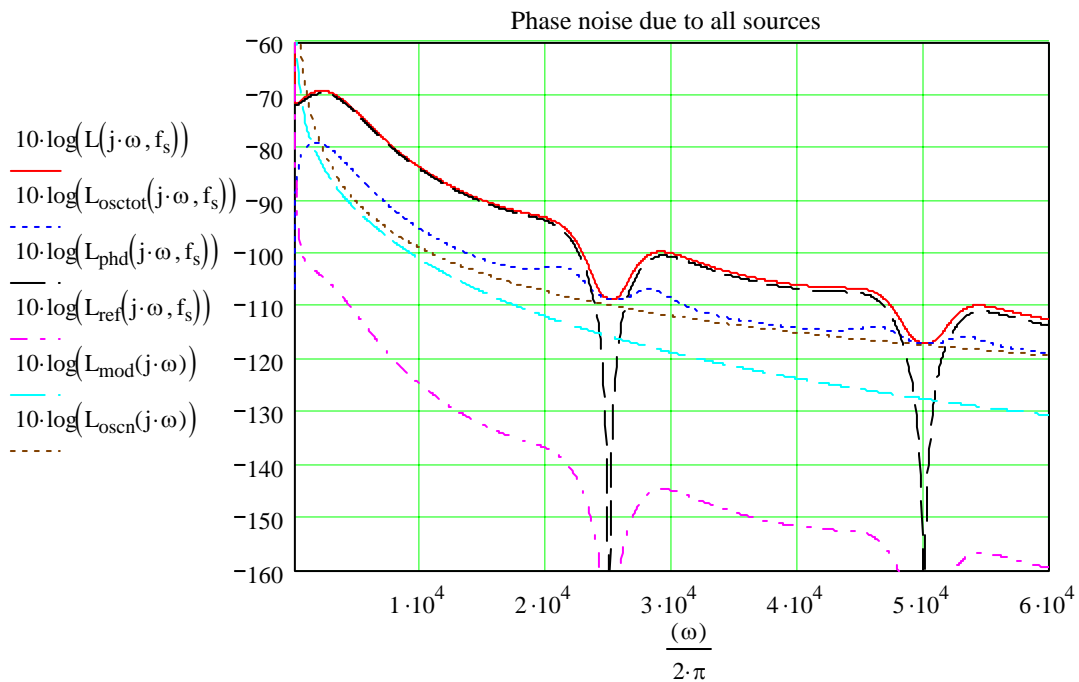
Plot shows the overall predicted noise for the PLL and the contributions from some of the relevant sources.



Plot below shows the measured phase noise of the test PLL. Note that the close in noise is higher than predicted by the simulation above. This is because the NTS1000A noise floor has been hit. The spec says the noise floor is -40dBc/Hz at 10Hz and -74dBc/Hz at 100Hz.



The same information re-plotted on a linear scale similar to what it might look like on a spectrum analyser. This also allows closer inspection of the noise in the nulls that occur at the sampling frequency and its harmonics



We can also make the oscillator noise higher than the reference/ phase detector/ divider noise and observe the result.

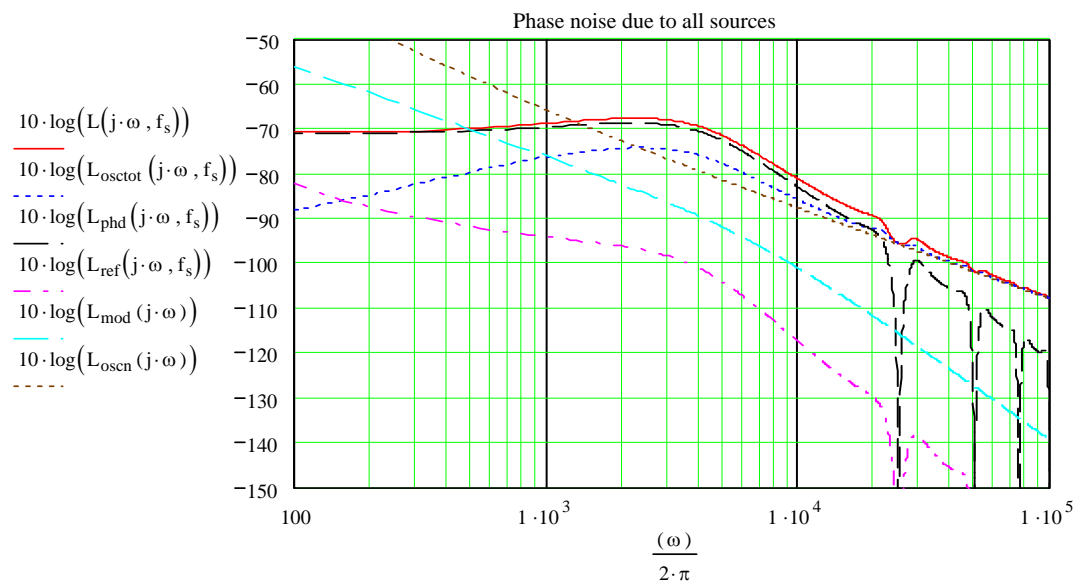
Here we leave the far-off phase noise and close in noise unchanged but raise the level of the 20dB/decade region by 20dB.

$$L_0 := -155 \quad f_0 := 3 \cdot 10^6$$

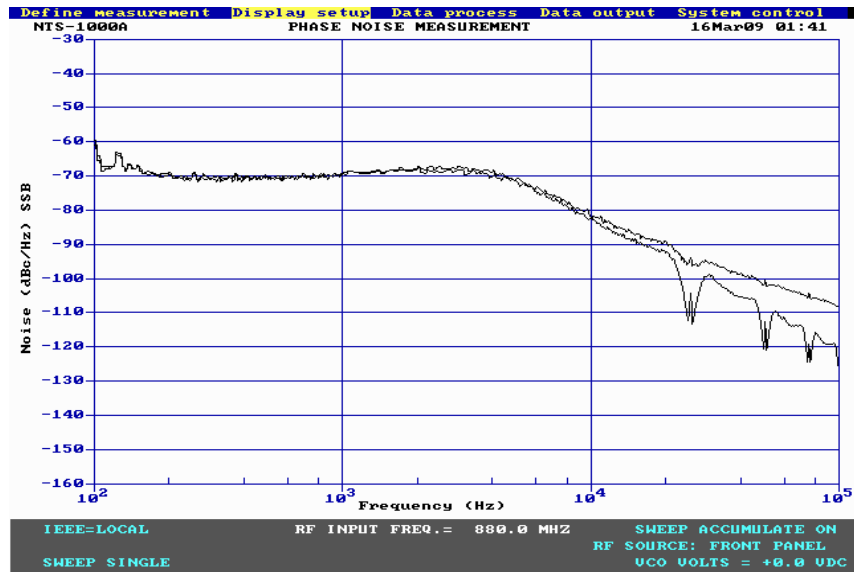
$$L_2 := -108 \quad f_2 := 100 \cdot 10^3$$

$$L_3 := -70 \quad f_3 := 1 \cdot 10^3$$

The plot below shows the effect of the increase in oscillator noise. Note that even though the oscillator noise is above the other noise sources the effects of sampling in the loop are evident at the harmonics of the sampling frequency.



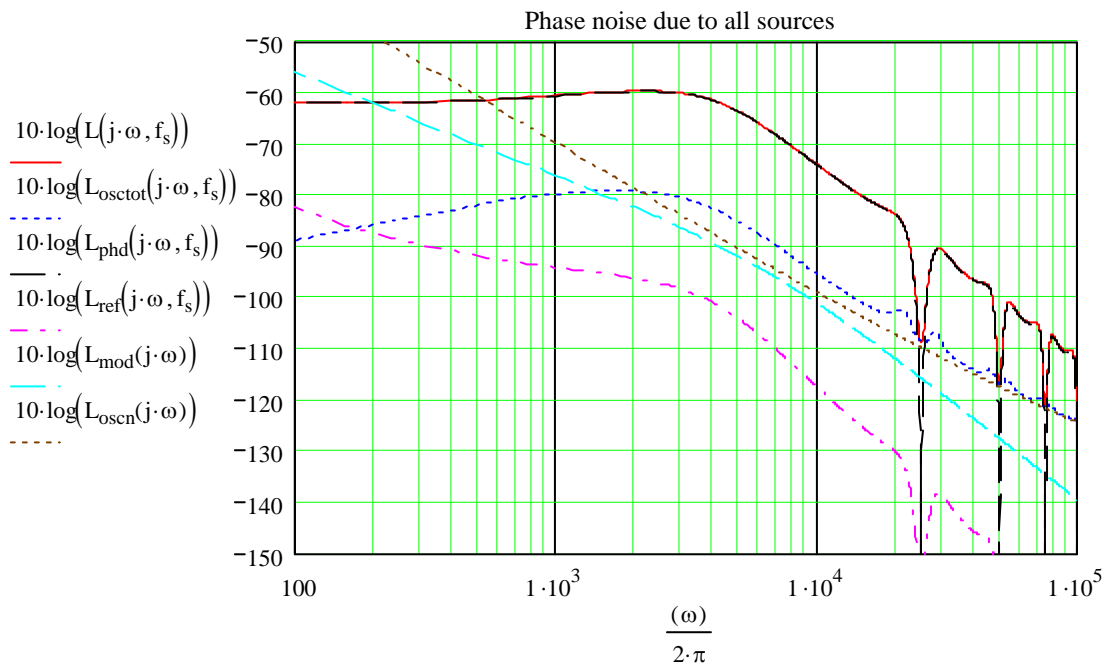
In the plot below the upper trace shows the measured effect of a relatively noisy oscillator. This is achieved by modulating the VCO with broad-band noise to produce the effect of a noisy oscillator. The lower trace shows the normal performance of the PLL.



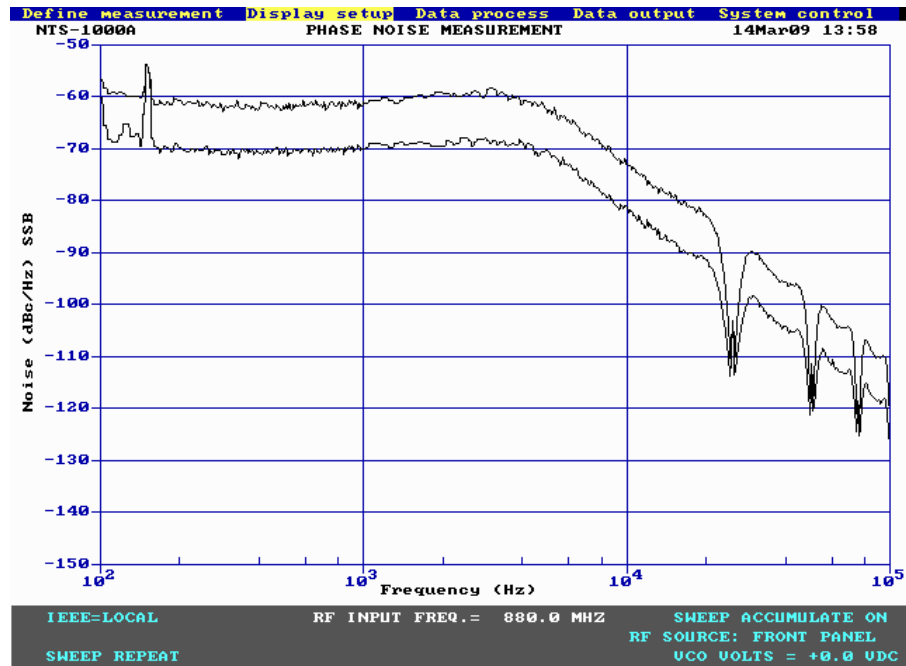
We can also make the Phase detector/ divider noise significantly higher than the oscillator noise and observe the result.

$$L_{pd} := -153 \text{ dB/Hz}$$

The plot below shows the effect of increasing the phase detector noise floor by 10dB. The same effect would be produced by a noisy reference divider or a noisy feedback divider. (Assuming the noise spectrum in each case is flat.)



In the plot below the upper trace shows the measured phase noise when the reference phase noise is raised by 10 dB by modulating the reference VCTCXO with pre-emphasised broadband noise. (To produce flat phase noise sidebands on the reference signal.) The lower trace shows the normal performance of the PLL.



Conclusions

There are a number of simulation packages provided by PLL chip manufacturers such as ADIsimPLL and EasyPLL, which although they may take discrete time effects into account for the prediction of transient responses, only use a linear approximation for the loop response and phase noise predictions. For many applications this simplification is sufficient but with the move toward fast settling PLLs where the loop bandwidth must be a significant fraction of the sampling frequency something closer to the true response is required.

This paper has attempted to provide a more accurate prediction of the loop response and the phase noise in a sampled PLL, and to back up the calculations with practical measurement of a test PLL. The measurements are in close agreement with the predicted results, giving a high level of confidence that the mathematics is correct and that predictions obtained by changing parameters such as reference noise, phase detector noise, VCO noise or sampling frequency will yield valid results.

Appendix

List of variable names:

ω	Angular frequency in radians/second
ω_s	Sampling frequency in radians/second
f_s	Sampling frequency in Hz
K_{pd}	Phase detector gain in mA/radian
K_{ovco}	Oscillator sensitivity in radians/second/volt
N	Loop divider ratio
R_p	Reference divider ratio
$R1$	Component values for loop filter
$C1$	Component values for loop filter
$C2$	Component values for loop filter
$G(s)$	Response of loop filter and phase detector
$K(s)$	Response of oscillator
$GK(s)$	Product of $G(s)$ and $K(s)$ which equates to the forward gain of the loop
$G_{ol}(s)$	Open loop gain for a continuous system
$Tr(s)$	Closed loop transfer response for continuous system
$G'_{ol}(s)$	Open loop gain for a sampled system
$Tr'(s)$	Closed loop transfer response for a sampled system
$G'_{ol}(s, f_s)$	Open loop gain for a sampled system
$Tr'(s, f_s)$	Closed loop transfer response for a sampled system
$L0(s)$	SSB phase noise 3MHz from the carrier, assumed to have f^0 distribution
$L2(s)$	SSB phase noise 100kHz from the carrier, assumed to have f^{-2} distribution
$L3(s)$	SSB phase noise 1kHz from the carrier, assumed to have f^{-3} distribution
$L_{oscn}(s)$	SSB phase noise model for the oscillator
$L'_{oscn}(s, f_s)$	SSB phase noise of the oscillator after sampling
$L'_{oscn1}(s, f_s)$	Contribution to the SSB phase noise of the oscillator due to sampling
$L_{osc1}(s)$	SSB phase noise due to the loops action on the $n = 0$ component of the oscillator noise
$L_{osc2}(s, f_s)$	SSB phase noise due to the loops action on the $n \neq 0$ component of the oscillator noise
$L_{osc}(s, f_s)$	SSB phase noise due to the loops action on the total oscillator noise
k	Boltsman's constant
T	Absolute temperature
$L_{mod}(s)$	SSB phase noise due to modulation of the VCO by voltage noise generated in the loop filter components
$L_{osceff}(s)$	Overall SSB phase noise due to the free running VCO and modulation noise due to thermal noise generated by the loop filter.
$L'_{osceff}(s, f_s)$	Sampled version of the SSB phase noise due to the free running VCO and modulation noise due to thermal noise generated by the loop filter.

$L'_{osceff1}(s,fs)$	Contribution to noise due to the sampling process, i.e. the total noise excluding the noise of a continuous system
$L_{osceff1}(s)$	Phase noise due to the loops action on the $n = 0$ component of overall oscillator noise
$L_{osceff1}(s)$	Phase noise due to the loops action on the $n \neq 0$ component of overall oscillator noise
$L_{osctot}(s,fs)$	Sampled sum of noise due to VCO and modulation
L_{pd_1Hz}	Phase detector noise referenced to 1Hz
L_{pd}	Phase detector noise at the sampling frequency of the PLL
$L_{phd}(s,fs)$	Contribution due to phase detector noise after the sampling action of the loop
$L_{x0}(s)$	SSB phase noise of VCTCXO 1MHz from the carrier, assumed to have f^0 distribution
$L_{x1}(s)$	SSB phase noise of VCTCXO 10kHz from the carrier, assumed to have f^{-1} distribution
$L_{x3}(s)$	SSB phase noise of VCTCXO 10Hz from the carrier, assumed to have f^{-3} distribution
$L_{tcxo}(s)$	SSB phase noise model of VCTCXO
$L_{ref}(s,fs)$	Phase noise of reference oscillator after sampling
$L'_{ref}(s,fs)$	Phase noise due to the reference oscillator after the sampling action of the loop
$L(s,fs)$	Oveall PLL output SSB phase noise due to all sources